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Dark Energy and Electrons

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Abstract We examine a shell model of the electron once rejected as being inconsistent, in the light of recent work related to dark energy and Casimir forces to show that indeed there is no inconsistency and a stable electron is indeed possible.

Keywords Dark energy · Shell · Stability · Electron

1 Introduction

We may reiterate that the "mysterious" background Dark Energy is identified with the quantum Zero Point Fluctuations in the background vacuum electromagnetic field, in recent years (cf. ref. [1] and references therein). This is indeed closely connected to the cosmology that has emerged since the 1998 observations by Perlmutter and others that demonstrated that, contrary to the earlier belief, the universe was actually accelerating, driven by the dark energy, that contributes a small cosmological constant (cf. ref. [2] and references therein). (Indeed this was predicted by the author's 1997 model. On the contrary the usual charge induced Zero Point Field leads to the cosmological constant that is 10^{120} orders of magnitude greater than what is observed.) The background Zero Point Field as pointed out by Wheeler [3] is a collection of ground state oscillators. The probability amplitude is

$$\psi(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{-(m\omega/2\hbar)x^2}$$

for displacement by the distance x from its position of classical equilibrium. So the oscillator fluctuates over an interval

$$\Delta x \sim (\hbar/m\omega)^{1/2}$$

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The background electromagnetic field is an infinite collection of independent oscillators, with amplitudes X_1, X_2 etc. The probability for the various oscillators to have amplitudes X_1, X_2 and so on is the product of individual oscillator amplitudes:

$$\psi(X_1, X_2, \dots) = e^{[-(X_1^2+X_2^2+\cdots)]}$$

wherein there would be a suitable normalization factor. This expression gives the probability amplitude ψ for a configuration $B(x, y, z)$ of the magnetic field that is described by the Fourier coefficients $X_1, X_2, \dots$ or directly in terms of the magnetic field configuration itself by, as is known,

$$\psi(B(x, y, z)) = Pe^{(-\int\int \frac{\mathbf{B}(\mathbf{x_1})\cdot\mathbf{B}(\mathbf{x_2})}{16\pi^3\hbar c r_{12}^2} d^3x_1 d^3x_2)}$$

P being a normalization factor. At this stage, we are thinking in terms of energy without differenciation, that is, without considering Electromagnetism or Gravitation etc as separate. Let us consider a configuration where the field is everywhere zero except in a region of dimension l, where it is of the order of $\sim \Delta B$. The probability amplitude for this configuration would be proportional to

$$e^{[-((\Delta B)^2 l^4/\hbar c)]}$$

So the energy of fluctuation in a region of length l is given by finally, the density [3, 4]

$$B^2 \sim \frac{\hbar c}{l^4}$$

So the energy content in a region of volume l^3 is given by

$$\beta^2 \sim \hbar c/l \tag{1}$$

As an alternative derivation, it is interesting to derive a model based on the theory of phonons which are quanta of sound waves in a macroscopic body [5]. Phonons are a mathematical analogue of the quanta of the electromagnetic field, which are the photons, that emerge when this field is expressed as a sum of Harmonic oscillators. This situation is carried over to the theory of solids which are made up of atoms that are arranged in a crystal lattice and can be approximated by a sum of Harmonic oscillators representing the normal modes of lattice oscillations. In this theory, as is well known the phonons have a maximum frequency ω_m which is given by

$$\omega_m = c\left(\frac{6\pi^2}{\vec{V}}\right)^{1/3} \tag{2}$$

In (2) c represents the velocity of sound in the specific case of photons, while $\vec{V} = V/N$, where V denotes the volume and N the number of atoms. In this model we write

$$l \equiv \left(\frac{4}{3}\pi\vec{V}\right)^{1/3}$$

l being the inter particle distance. Thus (2) now becomes

$$\omega_m = c/l \tag{3}$$

Let us now liberate the above analysis from the immediate scenario of atoms at lattice points and quantized sound waves due to the Harmonic oscillations and look upon it as a general set of Harmonic oscillators as above. Then we can see that (3) and (1) are identical as

$$\omega = \frac{mc^2}{\hbar}$$

Historically the original concept of the electron was that of a spherical charge distribution [6–8]. It is interesting to note that in the non-relativistic case, it was originally shown that the entire inertial mass of the electron equalled its electromagnetic mass. This motivated much work and thought in this interesting direction. To put it briefly, in non relativistic theory, we get [6],

$$\text{Kinetic energy} = (\alpha/2)\frac{e^2}{Rc^2}v^2$$

where R is the radius of the electron and α is a numerical factor of the order of 1. So we could possibly speak of the entire mass of the electron in terms of its electromagnetic properties.

It might be mentioned that it was still possible to think of an electron as a charge distribution over a spherical shell within the relativistic context, as long as the electron was at rest or was moving with a uniform velocity. However it was necessary to introduce, in addition to the electromagnetic force, the Poincare stresses—these were required to counter balance the mutual repulsive "explosion" of the different parts of the electron. The other is minimum spacetime intervals which are of the order of the Compton scale. The minimum spacetime interval removes, firstly the advanced field effects because this takes place within the Compton time and secondly the infinite self energy of the point electron disappears due to the Compton scale.

2 Quantum Mechanical Considerations

The Compton scale comes as a Quantum Mechanical effect, within which we have zitterbewegung effects and a breakdown of Causal Physics [9, 10]. Indeed Dirac had noted this aspect in connection with two difficulties with his electron equation. Firstly the speed of the relativistic Quantum Mechanical electron turns out to be the velocity of light. Secondly the position coordinates become complex or non Hermitian. His explanation was that in Quantum Theory we cannot go down to arbitrarily small spacetime intervals, for the Heisenberg Uncertainty Principle would then imply arbitrarily large momenta and energies. So Quantum Mechanical measurements are an average over intervals of the order of the Compton scale. Once this is done, we recover meaningful physics. All this has been studied afresh by the author more recently, in the context of a non differentiable spacetime and noncommutative geometry [4]. In Classical Physics the point electron leads to infinite self energy via the electromagnetic mass term e^2/R, where R is the radius which is made to tend to zero. If on the other hand R does not vanish, in other words we have an extended electron, then we have to introduce non electromagnetic forces like the Poincare stresses for the stability of this extended object, though on the positive side this allows the radiation damping or self force that is required by conservation laws.

Dirac could get rid of these problems by introducing the difference between the advanced and retarded potentials in his phenemenological equation in which the infinity was absorbed into a renormalized point particle mass: This was the content of the Lorentz Dirac equation.

The new term represents the radiation damping effect, but we then have to contend with the advanced potential or equivalently a non locality in time. However this non locality takes place within the Compton time, within which the electron attains a luminal velocity.

So the Lorentz Dirac equation on the other hand had unsatisfactory features like the derivative of the acceleration, the non locality in time and the run away solutions, features confined to the Compton scale.

The Feynman-Wheeler approach bypasses the infinity and the extended electron self force—but the mass is no longer electromagnetic. Moreover the net result is that there is only the desired retarded potential. But an instantaneous interaction with the rest of the charges of the universe is required. It is this interaction with the remaining charges which leads to the point electron's self energy. Surprisingly however the interaction with the rest of the charges in the immediate vicinity of the given charge in the Feynman-Wheeler formulation gives us back the Dirac antisymmetric difference with its non locality within the Compton scale. There is thus a reconciliation of the Dirac and the Feynman Wheeler approaches, once we bring into the picture, the Compton scale.

Outside this scale, the theory is causal that is uses only the retarded potential because effectively the advanced potential gets canceled out as it appears as the sum of the symmetric and antisymmetric differences.

The final conclusion was that in a Classical context a totally electromagnetic electron is impossible as also the concept of a point electron without introducing additional "unphysical" concepts including action at a distance. It was believed therefore that the electron was strictly speaking the subject of Quantum Theory.

Nevertheless in Dirac's relativistic Quantum Electron, we again encounter the electron with the luminal velocity within the Compton scale, precisely what was encountered in Classical Theory as well, as noted above. This again is the feature of a point space time approach. At this stage a new input was given by Dirac—meaningful physics required averages over the Compton scale, in which process, the unphysical zitterbewegung effects were eliminated. Nor has Quantum Field Theory solved the problem—one has to take recourse to renormalization, and as pointed out by Rohrlich, one still has a non electromagnetic electron. In any case, it appears that further progress would come either from giving up point space time or from an electron that is extended (or has a sub structure) in some sense [6–8, 11]. From this point of view the relativistic theory of the electron is inconclusive to date. As noted by Feynman himself in his famous Lectures on Physics (vol. II), "we do not know how to make a consistent theory—including the Quantum Mechanics—which does not produce an infinity for the self energy of the electron, or any point charge. And at the same time there is no satisfactory theory that describes a non-point charge ...". In the words of Hoyle and Narlikar [12], "... it was believed that the problem of the self force of the charge would not be solved except by recourse to Quantum Theory .... This hope has not been fully realized. Quantum Field Theory does alleviate the self energy problem but cannot surmount it without introducing the renormalization programme ...".

With the above background let us consider the stability of the electron that is formed by such a "condensation" [1] from a background. We consider the shell model of the electron that occupied the center stage for the early decades of the twentieth century. This model was much later re-examined by Casimir in the light of the possibility that the Casimir energy may provide Poincaré stresses required in the early model to retain the stability of the electron. To sum up Casimir had suggested that the background zero point field can manifest itself in the force of attraction between two thin metallic plates in vacuum. Based on these ideas Casimir suggested that a dense shell-like distribution of charge might partially (as in the Casimir Effect, generally), or even wholly, suppress vacuum fields in the interior of the shell

(Shell Models I and II, respectively). This could result in net inward radiation pressure from the EM (electromagnetic) vacuum fluctuation fields (playing the role of Poincare stresses) compensating outwardly-directed coulomb forces to yield a stable configuration at small dimensions. In QED renormalization theory this is handled by adding an infinite negative mass term to compensate the infinite positive coulomb term. Though such renormalization can be carried out in QED in an unambiguous and invariant way, from the standpoint of a semiclassical model it appears sufficiently ad hoc as to merit a search for an alternative. For Casimir's Shell Model II, which we now discuss, the charge density is taken to be sufficiently dense in a vanishing-radius shell so as to result in the total absence of interior vacuum fluctuation fields as a singularity is approached. The modeling for this case proceeds as follows:

1. Consider charge e to be homogeneously distributed on a spherical shell of (vanishing) radius $a \to 0$.
2. Under this assumption, the electric field E, given by

$$E = \frac{e}{4\pi\varepsilon_0 r^2} \tag{4}$$

leads to a (formally-divergent) coulomb energy

$$W_{coul} = \lim_{a\to 0}\int_a^\infty u_{coul}dV = \lim_{a\to 0}\int_a^\infty \frac{1}{2}\varepsilon_0 E^2 dV = \lim_{a\to 0}\left(\frac{e^2}{8\pi\varepsilon_0 a}\right) = \lim_{a\to 0}\left(\frac{\alpha\hbar c}{2a}\right) \tag{5}$$

where α is the fine structure constant, $\alpha = e^2/4\pi\varepsilon_0\hbar c \approx 1/137.036$.
3. With regard to the vacuum fluctuation electromagnetic fields, the spectral energy density given by

$$\rho(\omega)d\omega = \frac{\hbar\omega^3}{2\pi^2c^3}d\omega \tag{6}$$

leads to an associated divergent energy density

$$u_{vac} = \lim_{\Omega\to\infty}\left(\int_0^\Omega \frac{\hbar\omega^3}{2\pi^2c^3}d\omega\right) = \lim_{\Omega\to\infty}\left(\frac{\hbar\Omega^4}{8\pi^2c^3}\right) \tag{7}$$

where Ω constitutes an upper limit cutoff frequency that asymptotically approaches infinity. For Shell Model II in which an absence of interior vacuum fluctuation energy is assumed, the vacuum energy deficit inside the sphere is given by

$$W_{vac} = -u_{vac}V = \lim_{\Omega\to\infty, a\to 0}\left(-\frac{\hbar\Omega^4}{8\pi^2c^3}\frac{4}{3}\pi a^3\right) = \lim_{\Omega\to\infty, a\to 0}\left(-\frac{\hbar\Omega^4 a^3}{6\pi c^3}\right) \tag{8}$$

From (5) and (8) we therefore obtain for the Coulomb and vacuum energy contributions to the shell model

$$W = W_{coul} + W_{vac} = \lim_{a\to 0}\left(\frac{\alpha\hbar c}{2a}\right) - \lim_{\Omega\to\infty, a\to 0}\left(-\frac{\hbar\Omega^4 a^3}{6\pi c^3}\right) \tag{9}$$

4. We now require that the outwardly-directed Coulomb pressure, given by

$$P_{coul} = u_{coul} = \lim_{a\to 0}\left(\frac{\alpha\hbar c}{8\pi a^4}\right) \tag{10}$$

be balanced by the inwardly-directed vacuum radiation pressure [13]

$$P_{vac} = -\frac{1}{3}u_{vac} = \lim_{\Omega\to\infty}\left(-\frac{\hbar\Omega^4}{24\pi^2 c^3}\right) \tag{11}$$

Pressures (10) and (11) result in a (stable) balance at radius $a = a_b$ given by

$$a_b = \lim_{\Omega\to\infty}\left[\frac{(3\pi\alpha)^{1/4}c}{\Omega}\right] \to 0 \tag{12}$$

Under this pressure-balance condition, we have,

$$W = W_{coul} + W_{vac} = \lim_{a_b\to 0}\left(\frac{\alpha\hbar c}{2a_b}\right) - \lim_{a_b\to 0}\left(\frac{\alpha\hbar c}{2a_b}\right) \equiv 0 \tag{13}$$

and thus a corollary vacuum energy deficit serves to cancel the positive self-energy of the divergent Coulomb field. In other words, as stated at the beginning of point 4, that is (10), we achieve an exact balance between the outwardly directed Coulomb pressure (10) and the inwardly directed vacuum radiation pressure (11). This is exactly what is needed for stability of the electron now treated as a shell. Unlike previous attempts to develop such a model, the approach followed here does not force a specific (incorrect) value for the fine structure constant α.

3 Discussion and Conclusions

Thus, for Casimir's Shell Model II, the net contribution to the self-energy of the point-particle electron by the coulomb and vacuum fields vanishes. We are therefore led to conclude that, under the set of assumptions applicable to Casimir's Shell Model II, an inwardly-directed, divergent, electromagnetic vacuum fluctuation radiation pressure stably balances the divergent coulomb pressure. Furthermore, it does so in such a manner that, even in the limiting case of the point particle electron, no contribution to the self energy of the electron results from the divergent coulomb field. Thus a key requirement for the semiclassical electron model is met. As a result, to the degree that this result of the semiclassical analysis carries over to QED renormalization, it would appear that the additive infinite negative mass in the QED approach finds its source in a negative vacuum energy contribution as proposed in the Casimir model. Finally, the reality of high-energy-density vacuum fluctuation fields at the fundamental particle level is buttressed, while at the same time leading to a renormalization process compatible with a finite particle mass.

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